CKM-suppressed top quark decays $t \to q + W$ in the SM and beyond

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Top quark decays are of particular interest as a mean to test the standard model (SM) predictions, both for the dominant ($t \to b + W$) and rare decays ($t \to q + W, cV, cVV, c\phi^0, bWZ$). As the latter are highly suppressed, they become an excellent window to probe the predictions of theories beyond the SM. In particular, we evaluate the corrections from new physics to the CKM-suppressed SM top quark decay $t \to q + W$ ($q = d, s$), both within an effective model with right-handed currents and the MSSM. We also discuss the perspectives to probe those predictions at the ILC.

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I. INTRODUCTION.

After the discovery of the top quark at Fermilab Tevatron Collider [1, 2], experimental attention has been turned on the examination of its production mechanisms and decay properties. Within the Standard Model (SM), the top quark production cross section is evaluated with an uncertainty of the order of $\sim 15\%$, while it is assumed to decay to a W boson and a b quark almost 100% of the time. Due to its exceedingly heavy mass, the top quark is expected to be somehow related to new physics; it is also considered that the top quark may give some clue to understand the mechanism of electroweak symmetry breaking. Thus measuring its properties may serve as a window for probing physics beyond the SM [3].

The interactions of quarks and leptons, with gauge bosons, seem to be correctly described by the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory, as plenty of experimental data shows [4]. At tree-level, SM neutral interactions are diagonal, however, flavor changing neutral currents (FCNC) can arise at loop level. The fact that FCNC B-meson decays have been already detected, at rates consistent with the SM [5], represents a great success for the model itself. However, SM predictions for top quark related processes are strongly suppressed, although corresponding experimental bounds are rather weak. At the coming LHC it is important to study rare top quark decays, because about $10^7 - 10^8$ top pairs will be produced per year. Thus, rare decays with B.R. of order $10^{-5} - 10^{-6}$ may be detectable, depending on the signal. The presence of any hint for new top quark physics at LHC [6], would motivate further study to clarify the implications of those effects at the next generation of collider experiments [7].

In this paper we are interested in studying the CKM suppressed decays of the top quark, and the perspectives to detect them at the ILC [8]. It includes first a short review of top quark decays (section 2), then we discuss the expected branching ratio for the CKM-suppressed top quark decays $t \to q+W$ in the SM (section 3-A); here we also study how the extraction of the CKM element V_{ts} from B meson data, does depend on the assumptions made and in particular we determine how the value of V_{ts} changes when one includes a generic right-handed (RH) current, which could be incorporated using the effective lagrangian approach (section 3-B). This is a generic approach to new physics that serves to parametrize in a general setting corrections from new physics to flavor mixing [9]. Then, Section 4 includes the predictions for this decay from the Minimal SUSY extension of the SM (MSSM) [10]. The possibilities to detect such modes at the future International Linear Collider (ILC) is presented in section 5, while the conclusions and a summary of results are shown in section 6.

II. A SURVEY OF TOP DECAYS IN THE SM AND BEYOND

Because of the structure of the SM, the W boson coupling to fermion pairs ($td_iW^{\pm}$), is proportional to the CKM element V_{td_i}. Thus the decay $t \to b+W$ dominates its BR's. Radiative corrections to this mode have been evaluated in the literature, both in the SM and some extensions, mainly within the minimal SUSY extension of the SM (MSSM). In general, such corrections are at most of order 10%, and therefore difficult to detect at

BR	SM	THDM-III	MSSM
$\mathbf{BR(t \to sW)}$	2.2×10^{-3}	$\sim 10^{-3}$	$10^{-3} - 10^{-2}$
$\mathbf{BR(t \to c\phi^0)}$	$10^{-13} - 10^{-15}$	$\sim 10^{-2}$	$10^{-5} - 10^{-4}$
$\mathbf{BR(t \to c\gamma)}$	5×10^{-13}	$< 10^{-6}$	$< 10^{-7}$
$\mathbf{BR(t \to cZ)}$	1.3×10^{-13}	$< 10^{-6}$	$< 10^{-7}$
$\mathbf{BR(t \to cg)}$	5×10^{-11}	$< 10^{-6}$	$< 10^{-5}$
$\mathbf{BR(t \to c\gamma\gamma)}$	$< 10^{-16}$	$\sim 10^{-4}$	$< 10^{-8}$
$\mathbf{BR(t \to cWW)}$	2×10^{-13}	$10^{-4} - 10^{-3}$	??
$\mathbf{BR(t \to cZZ)}$	–	$10^{-5} - 10^{-3}$	??
$\mathbf{BR(t \to bWZ)}$	2×10^{-6}	$\simeq 10^{-4}$	??

TABLE I: Branching ratios for some CKM-suppressed and FCNC top quark decays in the SM and beyond, for $m_t = 173.5 - 178$ GeV. Decays into a pair of massive gauge bosons include finite width effects of final state unstable particles [18].

hadron colliders, but may be at the reach of the ILC.

FCNC top quark decays, such as $t \to c\gamma$, $t \to cg$, $t \to cZ$ and $t \to c\phi$ have been studied, for some time, in the context of both the SM and new physics [11]. In the SM, the branching ratio of FCNC top decays is extremely suppressed, as it is summarized in table I. The rare top quark decay $t \to c + \gamma$ was calculated first in ref. [12] in the SM and some extensions, the result implied a suppressed B.R., less than about 10^{-10}, which was confirmed when subsequent analysis [13] that included the correct top mass value and gave $BR(t \to c + \gamma) = 5 \times 10^{-13}$. The decays $t \to c + Z$ and $t \to c + g$ were also calculated in refs. [13]. The resulting branching ratios turned out to be $BR(t \to c + Z) = 1.3 \times 10^{-13}$ and $BR(t \to c + g) = 5 \times 10^{-11}$. None of them seem detectable at LHC nor at the ILC.

The top-charm coupling with the SM Higgs ϕ^0 could also be induced at one-loop level [14]. The resulting branching ratio is given by $BR(t \to c + \phi^0) = 10^{-15}$, which does not seem detectable neither. The FCNC top decays involving a pair of vector bosons in the final state, $t \to cVV$, can also be of interest [15]. Although one could expect such modes to be even more suppressed than the ones with a single vector boson, the appearance of an intermediate scalar resonance, as in the previous case, could enhance the B.R.. Furthermore, because of the large top quark mass, it also seems possible to allow the tree-level decay $t \to b + WZ$, at least close to threshold.

On the other hand, the top decay into the light quarks $t \to W + d(s)$ is suppressed, as they are proportional to $V_{td(s)}$. Furthermore, it is unlikely that these modes could be detected at all at hadron colliders. Probably for this reason, the SM corrections to this mode have not been studied, though the QCD corrections should be the similar for both modes. However, in extensions of the SM, it may be possible to get a large enhancement that could even make it detectable at the ILC, as will be shown in the next section.

Some typical results for the top decays in the SM are summarized in Table I. This table also includes, for comparison, the results for top branching ratios from models beyond the SM, in particular from the THDM-III [16] and SUSY [17], which will be discussed in what follows.

III. THE TOP QUARK DECAYS $t \to q + W$ IN THE SM AND WITH RH CURRENTS: CKM ANALYSIS

Although the decay $t \to s + W$ is expected to be suppressed in the SM, it is possible that new physics (e.g. LR models or SUSY) could induce an enhancement on this mode that could make it to be at the reach of ILC. This will have the attractive of allowing a tree-level determination of the CKM element V_{ts}, which otherwise need to be extracted from B-physics using one-loop induced processes, as we discuss next.

A. The decay $t \to s + W$ with standard CKM

Within the SM the rate of $t \to q + W$ decays at the tree-level is given by :

$$\Gamma^{SM}(t \to qW) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tq}|^2 \left\{ x^2(1 - 2x^2 + y^2) + (1 - y^2)^2 \right\} \times \lambda^{1/2}(1, x^2, y^2) \ , \tag{1}$$

where: $x = m_W/m_t, \ y = m_q/m_t$ $(q = d, s, b)$. The final quark masses can be safely neglected in all the relevant cases (even for $q = b$, $y^2 = (m_b/m_t)^2 = 8.4 \times 10^{-4}$ is negligible). Thus, the decay is proportional to the CKM elements V_{tq}, which are not so well known at present.

Here and thereafter we will assume $|V_{tb}| = 1$. Measurements of $b \to s\gamma$ by CLEO, BABAR and Belle, and its description in the SM framework give the following constraint on $|V_{ts}|$ [5]:

$$|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3} \ . \tag{2}$$

Assuming also top quark dominance of box diagrams contributions to ΔM_d in $B_d - \bar{B}_d$ mixing yields the following SM constraint [5]:

$$|V_{td}| = (7.4 \pm 0.8) \times 10^{-3} \ . \tag{3}$$

The corresponding branching fractions for the CKM-suppressed $t \to q'W$ $(q' = s, \ d)$ decays in the SM are:

$$\begin{aligned} B(t \to q'W) &= \frac{\Gamma(t \to q'W)}{\sum_q \Gamma(t \to qW)} \\ &\approx |V_{tq'}|^2 \\ &= \begin{cases} 1.65 \times 10^{-3}, \ \text{for } q' = s \ , \\ 5.5 \times 10^{-5}, \ \text{for } q' = d \ . \end{cases} \end{aligned} \tag{4}$$

Note that, in the case that the charged fermion couplings are modified by new physics, for instance as in the effective lagrangian approach [9] or the MSSM, to be discussed next, the values given in eq. (2,3) may not hold anymore. In particular, the dominance of the top quark in loop contributions to $b \to s\gamma$ and ΔM_{B_d} can be spoiled by the non-unitarity of the quark mixing matrix (as in ref. [9]) or other new physics effects. Note also that other indirect constraints on $V_{ts}, \ V_{td}$ can be obtained from the rare $K^+ \to \pi^+\nu\bar{\nu}$ decay, the ratio of mass differences of neutral B mesons, and from the exclusive $B \to \rho(K^*)\gamma$ decays. At present, theoretical and/or experimental uncertainties on these observables make them less competitive sources of information [5].

B. The decay $t \to W + s$ with RH top quark couplings

In this section we give an estimate of the width of top decay $t \to s + W$ that stems from the tsW coupling extracted from $b \to s\gamma$ decays, when a Left-Right modification is allowed. Namely, here we also study how the extraction of the CKM element V_{ts} from B meson data, does depend on the assumptions made and in particular we determine how the value of V_{ts} changes when one includes a generic right-handed (RH) current, which could be incorporated using the effective lagrangian approach.

We start from the analysis of the decay $B \to X_s\gamma$ as done by the authors in [19] who define the following ratios of Wilson coefficients (C_7, C_8) associated with the dipole operators O_7 and O_8,

$$r_7 = \frac{C_7(m_W)}{C_7^{SM}(m_W)} \ , \tag{5}$$

$$r_8 = \frac{C_8(m_W)}{C_8^{SM}(m_W)} \ , \tag{6}$$

$$r_7^R = \frac{C_7^R(m_W)}{C_7^{SM}(m_W)} \ , \tag{7}$$

$$r_8^R = \frac{C_8^R(m_W)}{C_8^{SM}(m_W)} \ , \tag{8}$$

while $C_{7,8}^R$ denote the Wilson coefficients of new dipole operators with opposite chirality to that of the Standard Model, and are evaluated at the scale m_W,

Then, we define the following useful ratio for the inclusive $B \to X_s\gamma$ decay,

$$\begin{aligned}
\frac{1}{N_{SL}} B(B \to X_s\gamma)|_{E_\gamma > (1-\delta)E_\gamma^{max}} &= B_{22}(\delta) + B_{77}(\delta)(|r_7|^2 + |r_7^R|^2) + B_{88}(\delta)(|r_8|^2 + |r_8^R|^2) \\
&\quad + B_{27}(\delta)Re(r_7) + B_{28}(\delta)Re(r_8) \\
&\quad + B_{78}(\delta)[Re(r_7 r_8^*) + Re(r_7^R r_8^{R*})] \ .
\end{aligned}$$

where $N_{SL} = B(B \to X_c e\bar{\nu})/0.105$ is a normalization factor to be determined from experiment.

The coefficients B_{ij} depend on the kinematical cut δ, with numerical values given in Ref. [19]. The δ parameter is defined by the condition that the photon energy be above a threshold given by $E_\gamma > E_0 = (1-\delta)E_\gamma^{max}$, where $E_\gamma^{max} = m_b/2$ is the maximum photon energy attainable in the parton model.

Next, considering the contributions from the Standard Model and from New Physics (RH couplings), we can write:

$$B(B \to X_s\gamma) = B^{SM}(B \to X_s\gamma) + B^{NP}(B \to X_s\gamma) \ . \tag{9}$$

In the case of the SM contributions we use the results of Neubert [20] which include the complete NNLO effects. For the NP contributions we choose the calculations of Kagan and Neubert [19] given above. If we identify $B(B \to X_s\gamma)$ with the experimental result, we can write the previous equation as follows:

$$B^{exp}(\delta) = |V_{ts}^* V_{tb}|^2 \left[\frac{B^{SM}(B \to X_s\gamma)|_{E_0=1.8 \ GeV}}{(0.0404^{+0.0016}_{-0.0006})^2} + \frac{X_{LR}}{(0.953 \pm 0.0195)^2} \cdot \frac{B(B \to X_c l\nu)}{0.105|V_{cb}|^2} \right] \tag{10}$$

where

$$X_{LR} = B_{77}(\delta)|r_7^R|^2 + B_{88}(\delta)|r_8^R|^2 + B_{78}(\delta)Re(r_7^R r_8^{R*}) \ , \tag{11}$$

and the SM prediction reads [20]

$$B^{SM}(B \to X_s\gamma)|_{E_0=1.8 \ GeV} = (3.38^{+0.31}_{-0.42} \pm 0.31) \times 10^{-4} \ . \tag{12}$$

In order to evaluate numerically the Wilson coefficients that enter in the definition of X_{LR}, eq.(11), we use the following expressions [21]: $C_7^{SM}(m_W) = F(x_t)$, $C_8^{SM}(m_W) = G(x_t)$, $C_7^R(m_W) = \xi(\frac{m_t}{m_b})\bar{F}(x_t)$, and $C_8^R(m_W) = \xi(\frac{m_t}{m_b})\bar{G}(x_t)$, where $F(x_t)$ and $G(x_t)$ are the Inami-Lim loop functions [22] and ξ denotes the mixing angle between the charged gauge bosons W_L and W_R of the left-right symmetric model. In the evaluation of the Inami-Lim functions we use $m_t = 174$ GeV.

Now, we consider the most recent experimental measurement of $B \to X_s\gamma$ reported by Belle (see for example, Ref. [23]):

$$B^{exp}(\bar{B} \to X_s\gamma)|_{E_0=1.8 \ GeV} = (3.38 \pm 0.30 \pm 0.29) \times 10^{-4} \ . \tag{13}$$

We can easily check that if NP contributions in eq. (10) are turned off, we recover the value of $|V_{ts}|$ given in eq. (2).

$\lvert\xi\rvert$	$\lvert V_{ts}\rvert$	$B(t \to s + W)$
0	40.4×10^{-3}	1.63×10^{-3}
0.01	34.9×10^{-3}	1.22×10^{-3}
0.02	26.3×10^{-3}	0.69×10^{-3}
0.03	20.0×10^{-3}	0.40×10^{-3}
0.04	15.9×10^{-3}	0.25×10^{-3}
0.05	13.1×10^{-3}	0.17×10^{-3}

TABLE II: Branching fractions for $t \to sW$ in the LR model by using the experimental $B \to X_s\gamma$ constraints.

Now, we can insert the experimental value of $B(B \to X_s\gamma)$ into eq. (10) and derive the modified values of $|V_{ts}|$ by allowing the parameter ξ to vary ($|\xi| \leq 0.05$). We will use $|V_{cb}| = 41.6 \times 10^{-3}$ [5] and $B(B \to X_c l\nu) = (10.75 \pm 0.16)\%$ [24]. The corresponding results are shown in Table II. In the same Table we also display the values of the $t \to sW$ branching ratios in the extension of the SM, with RH currents which were calculated according to the expression

$$B(t \to s + W) = |V_{ts}|^2(1 + \xi^2) \ . \tag{14}$$

Thus, we can conclude that the overall effect of the LR mixing angle ξ, is to decrease the prediction for $B(t \to s + W)$. This effect is sizeable and can eventually be discriminated at linear colliders. Conversely, if the $B(t \to s + W)$ is found to be in close agreement with the SM prediction, this would manifest into a very strong constraint on the LR mixing angle.

IV. TOP QUARKS DECAYS $t \to q + W$ IN THE MSSM

We shall discuss now top decays in the context of Supersymmetry (SUSY), which is studied mainly in connection with a possible solution of the hierarchy problem [10]. The Minimal Supersymmetric extension of the SM (MSSM), has well known attractive features, such as allowing gauge coupling unification and radiative electroweak symmetry breaking (EWSB). In addition SUSY also predicts tau-bottom Yukawa unification and provides a dark matter candidate. Within the MSSM, there are new sources of FCNC, and it may be difficult to satisfy current bounds on such transitions in a general SUSY breaking scheme. Generic entries for sfermion mass matrices produce so large FCNC that some mechanism must be found to suppress them, which is known as the SUSY flavour problem. Best known solutions to this problem include: *i) Degeneracy, ii) Decoupling and iii) Alignment* . In particular, the interactions of the gluino with squarks and quarks, could involve different families, which in turn could mediate FCNC transitions for quarks. Such gluino interactions can produce large enhancements on the top quark FCNC decays. In this section we shall discuss a particular ansatz that satisfies current FCNC constraints, and still allows a large stop-scharm mixing, which in turn could induce important corrections to the rare top quark decays.

A. The MSSM with large Stop-scharm mixing.

In the following we shall review the formalism presented in ref. [25], which shall be applied to evaluate the top decay $t \to s + W$, which has a larger branching ratio than $t \to d + W$.

Within the MSSM quark masses, as well as SUSY conserving F- terms, are contained in the superpotential of the model,

$$W = \lambda^u Q H_u U + \lambda^d Q H_d D \tag{15}$$

where $\lambda^{u,d}$ denote the 3×3 Yukawa matrices; Q, U, D are the superfields that denote the quark/squark doublets and singlets.

The soft-breaking squark-sector contains the following quadratic mass and trilinear A-terms,

$$\begin{aligned} &-\widetilde{Q}_i^\dagger (M^2_{\widetilde{Q}})_{ij}\widetilde{Q}_j - \widetilde{U}_i^\dagger (M^2_{\widetilde{U}})_{ij}\widetilde{U}_j - \widetilde{D}_i^\dagger (M^2_{\widetilde{D}})_{ij}\widetilde{D}_j \\ &+(A_u^{ij}\widetilde{Q}_i H_u \widetilde{U}_j - A_d^{ij}\widetilde{Q}_i H_d \widetilde{D}_j + \text{c.c.})\,, \end{aligned} \tag{16}$$

with $M^2_{\widetilde{Q},\widetilde{U},\widetilde{D}}$ and $A_{u,d}$ being 3×3 matrices in squark flavor-space. Here $\widetilde{Q}_i, \widetilde{U}_i, \widetilde{D}_i$ denote the squark doublet and singlets for family i. This gives a generic 6×6 mass matrix,

$$\widetilde{\mathcal{M}}^2_u = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^{2\,\dagger}_{LR} & M^2_{RR} \end{pmatrix}, \tag{17}$$

in the up-squark sector, where

$$\begin{aligned} M^2_{LL} &= M^2_{\widetilde{Q}} + M^2_u + \tfrac{1}{6}\cos 2\beta\,(4m_w^2 - m_z^2)\,, \\ M^2_{RR} &= M^2_{\widetilde{U}} + M^2_u + \tfrac{2}{3}\cos 2\beta \sin^2\theta_w\, m_z^2\,, \\ M^2_{LR} &= A_u v \sin\beta/\sqrt{2} - M_u\,\mu\cot\beta\,, \end{aligned} \tag{18}$$

with $m_{w,z}$ the masses of ($W^\pm$, Z^0) and M_u the up-quark mass matrix. For convenience, we will choose hereafter the super Cabibbo-Kobayashi-Maskawa (CKM) basis for squarks so that in (18), $A_u \to A_u' = K_{UL} A_u K^\dagger_{UR}$ and $M_u \to M_u^{\text{diag}}$, etc, with $K_{UL,R}$ denoting the rotation matrices for M_u diagonalization.

In our *minimal* Type-A scheme, we consider all large FCNCs to *solely* come from non-diagonal A_u' in the up-sector, and those in the down-sector to be negligible, i.e. we define at the weak scale,

$$A_u' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & y & 1 \end{pmatrix} A\,, \tag{19}$$

where x and y can be of $O(1)$, representing a naturally large flavor-mixing associated with $\tilde{t}-\tilde{c}$ sector. Such a minimal scheme of FCNC is compelling as it is consistent with all experimental data as well as the theoretical CCB/VS bounds [26]. Similar pattern may be also defined for A_d in the down sector, but the strong CCB/VS bounds permit $O(1)$ $\tilde{b}-\tilde{d}$ mixing only for very large $\tan\beta$, because $m_b << m_t$. Thus to allow a full range of $\tan\beta$ we consider an almost diagonal A_d. Moreover, identifying the non-diagonal A_u as the *only* source of observable FCNC phenomena for Type-A schemes implies that the squark-mass-matrices $M^2_{\widetilde{Q},\widetilde{U}}$ in eqs. (17)-(18) to be nearly diagonal. For simplicity we define

$$M^2_{LL} \simeq M^2_{RR} \simeq \tilde{m}0^2\,\mathbf{I}_{3\times 3}\,, \tag{20}$$

with $\tilde{m}0$ a common scale of scalar-mass.

Within this minimal Type-A scheme, we observe that the first family squarks $\tilde{u}_{L,R}$ decouple from the rest in (17) so that this 6×6 mass-matrix is reduced to a 4×4 matrix,

$$\widetilde{\mathcal{M}}^2_{ct} = \begin{pmatrix} \widetilde{m}_0^2 & 0 & 0 & A_x \\ 0 & \widetilde{m}_0^2 & A_y & 0 \\ 0 & A_y & \widetilde{m}_0^2 & -X_t \\ A_x & 0 & -X_t & \widetilde{m}_0^2 \end{pmatrix} \tag{21}$$

for squarks ($\tilde{c}_L$, $\tilde{c}_R$, $\tilde{t}_L$, $\tilde{t}_R$), where

$$\begin{aligned} &A_x = x\widehat{A}, \quad A_y = y\widehat{A}, \quad \widehat{A} = Av\sin\beta/\sqrt{2}\,, \\ &X_t = \widehat{A} - \mu\, m_t\cot\beta\,. \end{aligned} \tag{22}$$

In (21), we ignore terms suppressed by tiny factors of $O(m_c)$ or smaller. The reduced squark mass matrix (21) has 6 zero-entries in total and is simple enough to allow an exact diagonalization. Below, we summarize the general diagonalization of 4×4 matrix (21) for any $(x,\,y)$. The mass-eigenvalues of the eigenstates $(\tilde{c}_1,\,\tilde{c}_2,\,\tilde{t}_1,\,\tilde{t}_2)$ are,

$$\begin{aligned} M^2_{\tilde{c}1,2} &= \tilde{m}0^2 \mp \tfrac{1}{2}|\sqrt{\omega_+}-\sqrt{\omega_-}|\,, \\ M^2_{\tilde{t}1,2} &= \tilde{m}0^2 \mp \tfrac{1}{2}|\sqrt{\omega_+}+\sqrt{\omega_-}|\,, \end{aligned} \tag{23}$$

where $\omega_\pm = X_t^2+(A_x\pm A_y)^2$. From (23), we can deduce the mass-spectrum of stop-scharm sector,

$$M_{\tilde{t}1} < M_{\tilde{c}1} < M_{\tilde{c}2} < M_{\tilde{t}2}\,. \tag{24}$$

In Eq. (23), the stop $\tilde{t}_1$ can be as light as $120-300\,\mathrm{GeV}$ for the typical range of $\tilde{m}0^2 \simeq 0.5-1\,\mathrm{TeV}$. Then, the 4×4 rotation matrix of the diagonalization can be derived,

$$\begin{gathered} \begin{pmatrix} \tilde{c}_L \\ \tilde{c}_R \\ \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} = \begin{pmatrix} c_1c_3 & c_1s_3 & s_1s_4 & s_1c_4 \\ -c_2s_3 & c_2c_3 & s_2c_4 & -s_2s_4 \\ -s_1c_3 & -s_1s_3 & c_1s_4 & c_1c_4 \\ s_2s_3 & -s_2c_3 & c_2c_4 & -c_2s_4 \end{pmatrix} \begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \\ \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}, \\ s_{1,2} = \frac{1}{\sqrt{2}}\left[1-\frac{X_t^2\mp A_x^2\pm A_y^2}{\sqrt{\omega_+\omega_-}}\right]^{1/2}, \quad s_4=\frac{1}{\sqrt{2}}\,, \end{gathered} \tag{25}$$

and $s_3=0$ (if $xy=0$), or, $s_3=1/\sqrt{2}$ (if $xy\neq0$), where $s_j^2+c_j^2=1$, and $s_i=\sin\theta_i$, $c_i=\cos\theta_i$. The rotation (25) allows us to derive all relevant new Feynman rules in mass-eigenstates without relying on "mass-insertions".

Similar pattern may be also defined for A_d in the down-sector, but $O(1)$ mixings between $\tilde{b}$ and $\tilde{s}$ are allowed only for very large $\tan\beta$ by the strong CCB and VS bounds since $m_b \ll m_t$. To allow full range of $\tan\beta$ and focus our minimal scheme on the up-sector, we consider an almost diagonal A_d. Furthermore, since the large hierarchy $A_{33} >> A_{22} >> A_{11}$, is also enforced by CCB and VS bounds, we shall take $A_{33}=A_b\neq0$ and $A_{11}=A_{22}=0$, together with mass-degeneracy for the soft-breaking mass matrices as required by FCNC bounds. Then squark mixing only occurs for the third family, i.e. for $(\tilde{b}_L,\,\tilde{b}_R)$, which is described by the following 2×2 mass matrix,

$$\widetilde{\mathcal{M}}_d^2 = \begin{pmatrix} \widetilde{m}_0^2 & -X_b \\ -X_b & \widetilde{m}_0^2 \end{pmatrix} \tag{26}$$

where $X_b=-(A_b+\mu\tan\beta)m_b$. The mass eigenvalues for the eigenstates $(\tilde{b}_1,\,\tilde{b}_2\,)$ are,

$$M^2_{\tilde{b}1,2} = \tilde{m}0^2+\frac{1}{2}[\hat{\Delta}_b\mp\sqrt{\Delta_b+4X_b^2}]\,, \tag{27}$$

where $\Delta_b=\frac{\cos2\beta}{6}[2m_w^2+(1-2s_W^2)m_Z^2]$ and $\hat{\Delta}_b=-\frac{\cos2\beta}{6}[2m_w^2+(1+s_W^2)m_Z^2]$, and the mixing angle of the 2x2 diagonalization is given by:

$$s_b(c_b)=\frac{1}{\sqrt{2}}[1\mp\frac{\Delta_b}{\sqrt{\Delta_b^2+4X_b}}]^{1/2}. \tag{28}$$

B. SUSY corrections to the decay $t\to s+W$

Within the MSSM, the corrections to the dominant decay mode $t\to bW$ are of the order $1-10\%$; since $\Gamma(t\to b+W)\simeq1$ GeV, then one has that the size of SUSY corrections are of the order $.01-.1$ GeV. Now, one can estimate the SUSY corrections to the decay $t\to s+W$ by including an extra mixing angle $sin\theta_{ts}$

$\tan\beta$	$M_{\tilde{g}}$	K_{32}^{eff}	$r = K_{32}^{eff}/K_{32}^{0}$	$B(t \to s + W)$
5	300	0.043	1.14	5.06 $\times 10^{-3}$
	400	0.15	4.02	3.78 $\times 10^{-2}$
	500	0.79	21.27	9.98 $\times 10^{-1}$
	1000	0.067	1.79	9.25 $\times 10^{-3}$
20	300	0.046	1.23	5.53 $\times 10^{-3}$
	400	0.39	10.37	2.39 $\times 10^{-1}$
	500	0.23	6.37	9.15 $\times 10^{-2}$
	1000	0.076	2.05	8.19 $\times 10^{-2}$
50	300	0.047	1.26	5.69 $\times 10^{-3}$
	400	0.50	13.41	3.98 $\times 10^{-1}$
	500	0.19	5.12	5.99 $\times 10^{-2}$
	1000	0.078	2.10	1.19 $\times 10^{-2}$

TABLE III: SUSY corrections to the CKM-suppressed top vertex tsW.

(among stop and scharm, which will appear in the loops with gluino-squarks in the internal lines) , this mixing angle could be of order $.1 - .5$. Therefore the SUSY correction to the decay $t \to s + W$, would be in the range $10^{-4} - 10^{-2}$. Since $\Gamma(t \to s + W) \simeq 1.3 \times 10^{-3}$ GeV, and one has that the corrections could even be of order 100%, which can help to make it detectable at ILC.

We shall now present a detailed loop calculation within the context of the particular mixing scheme, where a large stop-scharm mixing is allowed, that was discussed in the previous subsection. The full radiatively corrected coupling tqW can be written as follows,

$$\begin{aligned} \Gamma^{\mu}_{tsw} &= -\tfrac{g_2}{\sqrt{2}}\gamma^{\mu} P_L K_{ts}^{eff} \,, \\ K_{ts}^{eff} &= K_{ts}^{0} + \Delta K_{ts}^{V} + \Delta K_{ts}^{S} \,, \end{aligned} \tag{29}$$

which includes the tree level vertex, which is proportional to the CKM element (K_{ts}^{0}), the corrections from stop-scharm-gluino triangle loops (ΔK_{ts}^{V}), as well as the stop- and scharm-gluino self-energy loops (ΔK_{ts}^{S}).

The result for the one-loop vertex corrections in Type-A model (with $x = y$) is

$$\Delta K_{ts}^{V} = \frac{\alpha_s K_{22}^{0}}{6\pi}(4\sin\theta_1\cos\theta_1)[-C_{24}(M_{\tilde{g}}^2, M_{\tilde{c}1}^2, M_{\tilde{b}}^2) - C_{24}(M_{\tilde{g}}^2, M_{\tilde{c}2}^2, M_{\tilde{b}}^2) + C_{24}(M_{\tilde{g}}^2, M_{\tilde{t}1}^2, M_{\tilde{b}}^2) + C_{24}(M_{\tilde{g}}^2, M_{\tilde{t}2}^2, M_{\tilde{b}}^2)] \tag{30}$$

where C_{24} are the 3-point C-function of Passarino-Veltman (PV), which depends on the gluino and squark masses. The Sin and Cos of the Mixing angle θ_1 are given in equation 16.

The dominant correction arising from the self-energy graphs (with $x = y$) can be expressed as:

$$\Delta K_{ts}^{S} = \frac{\alpha_s K_{22}^{0}}{6\pi}(\sin\theta_1\cos\theta_1)[-B_1(M_{\tilde{g}}^2, M_{\tilde{c}1}^2) - B_1(M_{\tilde{g}}^2, M_{\tilde{c}2}^2) + B_1(M_{\tilde{g}}^2, M_{\tilde{t}1}^2) + B_1(M_{\tilde{g}}^2, M_{\tilde{t}2}^2)] \tag{31}$$

where B_1 denotes the 2-point C-function of Passarino-Veltman (PV), which depends on the gluino and squark masses.

The results for the effective vertex K_{ts}^{eff} are shown in Table/Figure 2 for $X = 0.9$, $m_0 = 500$ GeV and several values of $\tan\beta$ and gluino mass. We notice that the correction can be larger than the tree-level result by about one order of magnitude. Thus, one could have $BR(t \to sW)$ even of order 10^{-1}, which looks promising to be detectable at the future ILC.

V. DETECTION OF TOP DECAYS AT THE ILC

Direct measurement of $|V_{td}|$ or $|V_{ts}|$ are not possible at LHC. We will explore then the possibility of detecting top transition to light quarks with the next generation lepton collider experiments.

The International Linear Collider (ILC) is a proposed electron-positron collider whose design is being addressed in the context of the Global Design Effort [27]. ILC has been agreed in a world-wide consensus to be the next

large high energy physics experimental facility. The nature of the ILC's electron-positron collisions would give it the capability to answer compelling questions that, discoveries at the LHC, will raise from the identity of dark matter to the existence of extra dimensions. The overall system design [28] has been chosen to realize the physics requirements with a maximum center of mass energy of 500 GeV and a peak luminosity of $2 \times 10^{34} cm^{-2} s^{-1}$. The total footprint is around 31 km; two facing linear accelerators, main linacs, will accelerate electron and positron beams from their injected energy of 15 GeV to the final beam energy of 250 GeV or less, over a combined lenght of 23 km. The electron source, the damping rings, and the positron auxiliary ('keep-alive') source are centrally located around the interaction region (IR). The plane of the damping rings is elevated by 10 m above that of the beam delivery system to avoid interference. The baseline configuration furthermore foresees the possibility of an upgrade to energies of about 1 TeV. In order to upgrade the machine to E_{cms}= 1 TeV, the linacs and the beam transport lines from the damping rings would be extended by another 11 km each. While in hadron collisions, it is technically feasible to reach the highest centre-of-mass energies, often usefull in order to discover new particles, in e^+e^- collisions the highest precision for a measurement can be achieved. High-precision physics at the ILC is made possible in particular by the collision of point-like objects with exactly defined initial conditions, by the tunable collision energy and by the possibility of working with polarized beams.

The present phenomenological work have been performed assuming a center of mass energy of $\sqrt{s} = 500$ GeV and an integrated luminosity, considering two running experiments, taking data at the same time, for about 4 years, plus the year 0, with a total integrated luminosity of 1 ab^{-1} [29]. Events are generated using the PANDORA-PYTHIA Montecarlo generator and applying a parametric detector simulation. Heavy quarks, $b-$jets and $c-$jets, are tagged using their well known unambiguous properties such as their mass and their long lifetimes. To tag light-quark jets is much more difficult but anyhow this is needed in order to get meaningful measurement of the CKM matrix elements. The technique used, in the present work, is the so called large flavour tagging method (LFTM) [30]. Particles, with large fraction $x_p = 2p/E_{cm}$ of the momentum, carry information about their primary flavour. Then it is possible to define a class of functions: $\eta_q(x_p)$ that represent the probability, for a quark of a flavour q, to develop into a jet in which i is the particle that brings the largest $x_p = 2p/E_{cm}$. Tagging efficiencies can be extracted with almost no reliance on the hadronization model, using a sample of Z^0, by evaluating single and double tag probability functions. In order to simplify the calculations, hadronisation symmetries have been introduced. In particular we assume that: $\eta_d^{\pi^\pm} = \eta_u^{\pi^\pm}$, $\eta_s^{K^\pm} = \eta_s^{K^0}$, $\eta_d^{e^\pm} = \eta_u^{e^\pm}$, $\eta_d^{\Lambda(\bar{\Lambda})} = \eta_u^{\Lambda(\bar{\Lambda})}$ and so on. Within the SM, a top, with a mass above Wb threshold, is predicted to have a decay width dominated by the two-body process: $t \to Wb$ $(Br \simeq 0.998)$ [5]. Then a $t\bar{t}$ pair will mainly decay into $WbWb$. As we are interested to study the CKM-suppressed top quark decay $t \to sW$, we will look for $t\bar{t} \to WsWb$ and $t\bar{t} \to WsWs$ that are available with a probability of $2(1-\mathrm{Br}_{t\to Ws})\mathrm{Br}_{t\to Ws} \simeq 2\mathrm{Br}_{t\to Ws}$ and $\mathrm{Br}^2_{t\to Ws}$ respectively. This decay modes can be classified according to their topological final states as follow: $i)$ dilepton mode, where both W decays are leptonic, with 2 jets arising from the quark hadronization (b or s) and missing transverse energy ($\not{E}_\mathrm{T}$) coming from the undetected neutrinos; $ii)$ Lepton+jets mode, where one W decays leptonically and the other one into quarks, with 4 jets and $\not{E}_\mathrm{T}$; $iii)$ All jets mode, where both the W's decay into quarks with 6 jets and no associated $\not{E}_\mathrm{T}$. In our analysis, we searched for top dilepton decay candidates without considering τ leptons in the final state. Events were asked to have two high-p_T leptons, and a reconstructed dilepton invariant mass $M_{\ell^+\ell^-}$ outside the Z^0 mass window. The jet-tagging requirements were: one tagged $b-$jet, vetoing the presence of an extra $b-$jet and also vetoing the presence of a tagged $c-$jet. The discrimination of $s-$jets from other light-quark-jets have been achieved by using the large flavour tagging technique. By assuming an ILC integrated luminosity of 1 ab^{-1}, and two running experiments at the same time, we found that a branching ratio sensitivity up to 10^{-3} for the process $t \to sW$ is achievable. This makes possible to search for physics beyond the SM in the top decay channel $t \to s\,W$.

VI. CONCLUSIONS AND PERSPECTIVES

Rare decays of the top quark can be interesting probes of new physics. Within the SM one has $BR(t \to s + W) \simeq 1.5 \times 10^{-3}$. When one allows RH top currents the CKM element gets modified in such a manner that the B.R for the decay $t \to sW$ gets supressed. In the MSSM, we can get an enhancement of order 50%, which may help to make it detectable at ILC. Assuming a center of mass energy of $\sqrt{s} = 500$ GeV, and a total integrated luminosity of 1 ab^{-1} at ILC, it will be possible to reach sensitivity for BR's up to 10^{-3}. Thus we

can summarize our results as follows:

1. Rare decays of the top quark can be interesting probes of new physics.
2. $B.R.(t \to s + W) \simeq 1.5 \times 10^{-3}$ is reached in the SM.
3. Assuming a RH top current results in a suppression of V_{ts} and therefore in $BR(t \to sW)$.
4. In the MSSM, we can get an enhancement of order 50 percent, which may help to make it detectable at ILC.
5. Thus, the future ILC will allow us to complete our understanding of top quark physics, by measuring the CKM suppressed decay $t \to sW$.

VII. ACKNOWLEDGMENTS

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